NUMERICAL METHOD FOR DETERMINATION OF GENERALIZED ANGULAR COEFFICIENTS

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A method is proposed for numerical determination of generalized angular coefficients for arbitrary surfaces. Examples of calculations are presented.

In solving radiant heat exchange problems with a radiating and an absorbing gas layer by the zonal method, determination of the geometrical optical characteristics of the thermal radiation is of great importance. The presence of a quite simple method for determination of generalized angular coefficients permits a significant reduction in volume of the task of filling the matrix coefficients for unknowns in the zonal equation system [1]. The existing methods, using the function Ki(x) permit finding generalized angular coefficients for a limited number of surfaces [2, 3]; use of the Monte Carlo method requires a large number of tests, and thus, a large amount of computer time [4, 5].

In the present study, to calculate generalized angular coefficients from the area 1 to the area 2 we use the expression [1]

$$\psi_{A_1-A_2}=\frac{1}{A_1}\int\limits_{A_1}\int\limits_{A_2}\exp\left(-K_g\rho\right)\frac{\cos\alpha_1\cos\alpha_2}{\pi\rho^2}\,dA_1dA_2. \tag{1}$$

Let the surfaces A_1 and A_2 be specified by the equations

$$z_1=f_1(x_1,\ y_1),\ z_2=f_2(x_2,\ y_2). \tag{2}$$

We will consider the vector

$$\vec{\rho}=((x_2-x_1),\ (y_2-y_1),\ (z_2-z_1)) \tag{3}$$

and the normal vector to the surface

$$\vec{n}=\left(-\frac{\partial f}{\partial x},\ -\frac{\partial f}{\partial y},\ 1\right). \tag{4}$$

Then

$$\cos\alpha=\frac{\vec{n}\cdot\vec{\rho}}{|\vec{n}|\cdot|\vec{\rho}|},\quad -\frac{\pi}{2}\leqslant\alpha\leqslant\frac{\pi}{2},\quad \cos\alpha\geqslant 0, \tag{5}$$

$$dA=\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2+1}\;dxdy. \tag{6}$$

Finding $\cos\alpha_1$, $\cos\alpha_2$, dA_1, dA_2 with Eqs. (5), (6) and substituting the expressions found in Eq. (1), we obtain

$$\psi_{A_1-A_2}=\frac{1}{\pi A_1}\int\limits_{A_1}\int\limits_{A_2}\exp\left(-K_g\rho\right)\cdot F(x_1,\ y_1,\ x_2,\ y_2)\,dx_1dy_1dx_2dy_2, \tag{7}$$

where

$$F(x_1,\ y_1,\ x_2,\ y_2)=\left[-(x_2-x_1)\frac{\partial f_1}{\partial x_1}-(y_2-y_1)\frac{\partial f_1}{\partial y_1}+(f_2-f_1)\right]\left[-(x_2-x_1)\frac{\partial f_2}{\partial x_2}-(y_2-y_1)\frac{\partial f_2}{\partial y_2}+(f_2-f_1)\right]\frac{1}{\rho^4}; \tag{8}$$

$$\rho=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(f_2-f_1)^2}.$$

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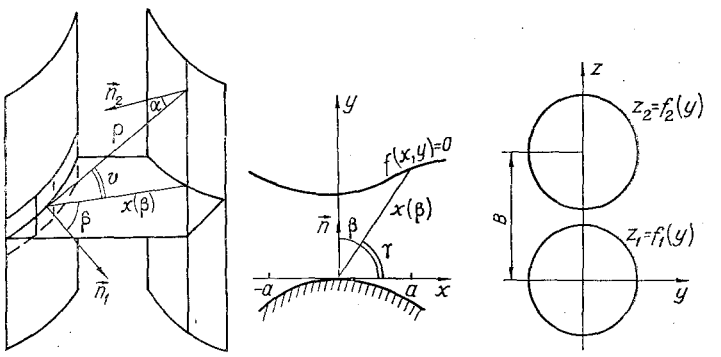

Fig. 1. Determination of generalized angular coordinates.

In the case of heat exchange between cylindrical areas with parallel directrices $z_1 = f_1(y_1)$, $z_2 = f_2(y_2)$, the integrand function F takes on the form

$$F(x_1, y_1, x_2, y_2) = \frac{1}{\rho^4}\left[-(y_2-y_1)\frac{df_1}{dy_1} + (f_2-f_1)\right]\left[-(y_2-y_1)\frac{df_2}{dy_2} + (f_2-f_1)\right]. \tag{9}$$

The fourfold integrals thus obtained are calculated approximately by Simpson's method, with automatic step selection [6]. In the case where the projections of the surfaces A_1 and A_2 on the plane xy A_1^0 and A_2^0 are not rectangles, integrals with variable upper limits are obtained. These can be reduced to integrals with constant limits, if for the integration area we take a rectangle including within itself A_1^0 and A_2^0, and take the integrand function in the following form:

$$\varphi = \begin{cases} \exp(-K_g\rho)\,F(x_1, y_1, x_2, y_2), & (x_1, y_1)\in A_1^0,\ (x_2, y_2)\in A_2^0, \\ 0, & (x_1, y_1)\bar{\in} A_1^0,\ (x_2, y_2)\bar{\in} A_2^0. \end{cases} \tag{10}$$

To calculate generalized angular coefficients from an elementary area, which is part of a cylindrical surface, onto an infinite cylindrical surface with parallel directrix, it is convenient to use the expression [2]

$$\psi_{dA_1-A_2} = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\beta_1}^{\beta_2}\exp(-K_g\rho)\cos\beta\cos^2\vartheta\,d\beta\,d\vartheta. \tag{11}$$

The angles β, ϑ and the ray length ρ are shown in Fig. 1. To determine the ray length we use the expression

$$\rho = X(\beta)/\cos\vartheta, \tag{12}$$

where $X(\beta)$ is the distance from the elementary area to the cylindrical surface in the plane perpendicular to the surface (see Fig. 1).

To find $X(\beta)$ it is necessary to write the equation for the cylindrical surface in a coordinate system fixed to the elementary area (ordinate directed along normal to this area):

$$f(x, y) = 0. \tag{13}$$

Adding the equation of a ray departing from the origin of the coordinate system, we obtain a system of two equations, permitting determination of $X(\beta)$:

$$f(x, y) = 0,\ y = kx, \tag{14}$$

where $k = \tan\gamma = \cot\beta$ is the angular coefficient of the straight line (see Fig. 1).

The ray length can then be found from

$$X(\beta) = \sqrt{x_0^2 + y_0^2}, \tag{15}$$

where x_0, y_0 are the solutions of Eq. (14) smallest in absolute value, which can be obtained analytically or by numerical methods.

As an example of calculation of $\psi_{dA_1-A_2}$ we will consider two cases: radiation of an elementary area onto a plane surface and onto the internal and external surfaces of a right circular cylinder, as is most often met

TABLE 1. Comparison of Data of Various Authors

h, m	ψ		
	[7]	[8]	present study
0,5	0,542	0,53	0,523
1,0	0,438	0,42	0,415
1,5	0,368	0,35	0,350
2,0	0,332	0,32	0,314

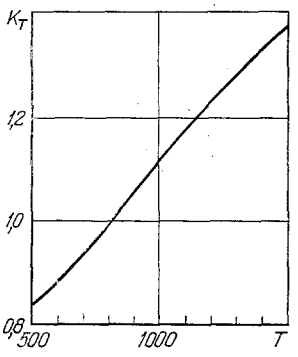

Fig. 2

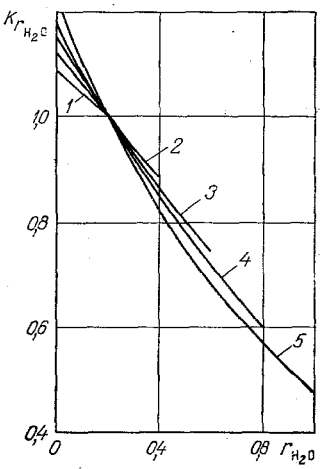

Fig. 3

Fig. 2. Correction coefficient K_T versus temperature, T, °C.

Fig. 3. Correction coefficient K_{rH_2O} versus volume fraction of H_2O vapor (all quantities are dimensionless); 1) $P_\Sigma/P_0 = 0.2$; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.0.

with in solving problems of heating of complex charges in heaters and furnaces. The distance $X(\beta)$ has an exact analytic expression in each of these cases.

1. Radiation of elementary area onto plane surface. The formula for determination of $X(\beta)$ has the form

$$X(\beta) = \frac{h}{\cos\beta}, \tag{16}$$

when the surface and area are parallel, and

$$X(\beta) = \frac{h}{\sin\beta}, \tag{17}$$

when the surface and area are perpendicular.

2. Radiation of elementary area onto internal and external surface of cylinder. To determine the distance $X(\beta)$ the equations of a circle and straight line are solved simultaneously:

$$(x - x_B)^2 + (y - y_B)^2 = R^2, \quad y = kx. \tag{18}$$

The coordinates of the center of the second circle x_B and y_B are determined in a coordinate system fixed to the elementary area, with ordinate directed along the normal to this area.

The quantity $X(\beta)$, in the case of radiation onto the external cylinder surface, is defined as:

$$\begin{aligned} X(\beta) &= |X|_{\min}\sqrt{k^2+1}, \quad \beta \neq 0, \\ X(\beta) &= |y_B - \sqrt{|-x_B^2 + R^2|}|, \quad \beta = 0, \end{aligned} \tag{19}$$

where $|X|_{\min}$ is the absolute value of the smaller of the two solutions of Eq. (18).

In the case of radiation onto the internal cylinder surface,

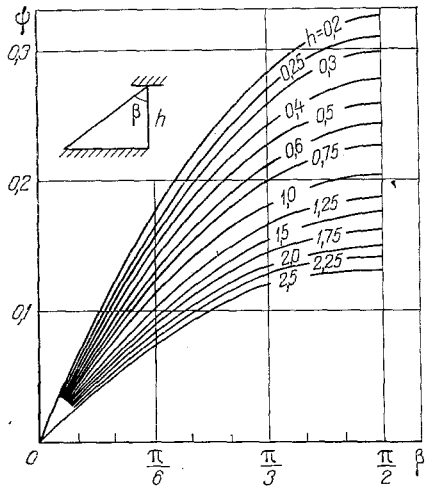

Fig. 4. Nomogram for finding generalized angular coefficients (β, rad; ψ, dimensionless).

$$X(\beta)=\frac{y_B+\frac{x_B}{k}\sqrt{\left|-\left(\frac{y_B}{k}-x_B\right)^2+R^2\left(1+\frac{1}{k^2}\right)\right|}}{\sqrt{1+\frac{1}{k^2}}},\quad \beta\neq 0,\ \beta\neq\frac{\pi}{2},$$

$$X(\beta)=|y_B+\sqrt{|-x_B^2+R^2|}|,\quad \beta=0, \tag{20}$$

$$X(\beta)=|x_B+\sqrt{|-y_B^2+R^2|}|,\quad \beta=\frac{\pi}{2}.$$

To determine generalized angular coefficients a FORTRAN program using Eqs. (7), (8), (11), (12) was written. To verify the technique the results of the calculations for radiation of an elementary area onto a plane surface (using Eq. (16)) were compared with data presented in [7, 8].

Table 1 presents the calculation results for various distances between the surfaces. A comparison of the data will indicate the satisfactory accuracy of the calculations for engineering problems.

Calculations of generalized angular coordinates were performed for the radiation of several geometric forms for a gas medium consisting of mixed CO_2 and H_2O, without consideration of radiation selectivity. The calculations were carried out to an accuracy of $\varepsilon = 0.005$, in connection with which the step in numerical integration was chosen automatically. In all cases considered, 16 nodes in each variable were sufficient to achieve the desired accuracy. Time for calculation of one coefficient with an ES 1020 computer did not exceed 1.5-2 min.

Calculations were performed for $T^0 = 800°C$, $r^0_{H_2O} = 0.2$; P_Σ/P_0 took on values of 0.2, 0.4, 0.6, 0.8, 1.0. In order to determine values of $\psi_{A_1\text{-}A_2}$ for parameters differing from the basic ones, the dependence of $\psi_{A_1\text{-}A_2}$ on temperature and on volume concentration of H_2O was found for various P_Σ/P_0. Results of the calculations are presented in Figs. 2, 3 in the form

$$K_T=\frac{\psi_{A_1-A_2}(T_g)}{\psi_{A_1-A_2}(T_g^0)}=f(T_g),\quad K_{r_{H_2O}}=\frac{\psi_{A_1-A_2}(r_{H_2O})}{\psi_{A_1-A_2}(r^0_{H_2O})}=\varphi(r_{H_2O}).$$

To determine the quantity $\psi_{A_1\text{-}A_2}$ at parameter values differing from the basic ones, it is necessary to have the value of $\psi_{A_1\text{-}A_2}$ obtained for T^0_r, $r^0_{H_2O}$, multiplied by the corresponding correction coefficients.

As an example of application of the proposed method, a calculation of generalized angular coordinates was made, where A_2 is an infinite cylindrical surface and A_1 is a portion of a right circular cylinder, the directrix of which is parallel to the directrix of A_2 and is one unit long, and for a rectangle, which is parallel to the directrix of A_2, with one side one unit long. The surfaces are specified by the equations (see Fig. 1)

$$z_1=\sqrt{1-y_1^2}\ \text{where}\ z_1=0,\ z_2=B-\sqrt{1-y_2^2},\ B>2. \tag{21}$$

Finding $\partial z_1/\partial y_1$ and $\partial z_2/\partial y_2$, we obtain the following expression for generalized angular coordinates in the first case:

TABLE 2. Dependence of Generalized Angular Coordinates on Dimensions and Relative Position of Surfaces

a, m	B, m					
	2,5		3,0		3,5	
	cylinder	plane	cylinder	plane	cylinder	plane
0	0,432	0,432	0,246	0,246	0,174	0,174
0,1	0,413	0,415	0,229	0,230	0,160	0,160
0,3	0,384	0,410	0,218	0,227	0,154	0,159
0,5	0,330	0,392	0,196	0,223	0,143	0,158

$$\psi_{A_1-A_2}=\frac{1}{2\pi\arcsin a}\int_{-a}^{a}\int_{-1}^{1}\int_{-\infty}^{\infty}\exp(-K_g\rho)\,F\,dy_1dy_2dx_2,\quad F=\left\{\left[\frac{y_1(y_2-y_1)}{\sqrt{1-y_1^2}}+B-\sqrt{1-y_1^2}-\sqrt{1-y_2^2}\right]\times\right. \tag{22}$$

$$\left.\times\left[-\frac{y_2(y_2-y_1)}{\sqrt{1-y_2^2}}+B-\sqrt{1-y_1^2}-\sqrt{1-y_2^2}\right]\right\}\Big/\{[x_2^2+(y_2-y_1)^2+(B-\sqrt{1-y_1^2}-\sqrt{1-y_2^2})^2]^2\}.$$

In the calculations it is necessary that

$$\frac{y_1(y_2-y_1)}{\sqrt{1-y_1^2}}+B-\sqrt{1-y_1^2}\sqrt{1-y_2^2}\geqslant 0,$$
$$-\frac{y_2(y_2-y_1)}{\sqrt{1-y_2^2}}+B-\sqrt{1-y_1^2}-\sqrt{1-y_2^2}\geqslant 0. \tag{23}$$

In a similar manner we obtain the value of $\psi_{A_1-A_2}$, when A_1 is a rectangle.

Calculation results for various values of a and B are presented in Table 2. Calculations at $a = 0$ were performed with Eq. (11). A comparison of the results with $a = 0$ and not equal to 0 shows how much the generalized angular coordinates differ in the cases where A_1 is an elementary area and in the case of a real form A_1. As follows from the table, this divergence increases with increase in the size of area A_1.

Results of calculating the dependence of generalized angular coefficients from an elementary area dA_1 onto a plane surface on angle β are shown in Fig. 4.

To determine $\psi_{dA_1-A_2}$ it is necessary to find the angles β_1 and β_2, starting from the relative position of the surfaces, after which the nomograms are used to find two values of $\psi(\beta_1)$ and $\psi(\beta_2)$. If the angles are on the same side of the normal, then $\psi = |\psi(\beta_1) - \psi(\beta_2)|$, while if they are on different sides, then $\psi = \psi(\beta_1) + \psi(\beta_2)$.

In conclusion, it should be noted that the algorithms and programs developed permit a significant reduction in the volume of labor in finding generalized angular coefficients. To perform the calculations only the most general data on geometric position of the surfaces, temperature, and composition of the radiating gas need be specified to the computer.

NOTATION

$\psi_{A_1-A_2}$, generalized angular coefficient from area A_1 to area A_2; A, area of elemental area, m^2; K_g, absorption coefficient of gas, m^{-1}; ρ, ray length, m; α, angle between direction of ray and normal to surface, rad; $\pi \approx 3.141$; h, distance from elementary area to plane surface, m; R, radius of cylinder forming surface, m; r_{H_2O}, volume fraction of water vapor in gaseous medium (at atmospheric pressure equal to P_{H_2O}/P_0); $P_\Sigma = P_{CO_2} + P_{H_2O}$, total partial pressure of CO_2 and H_2O, N/m^2; $P_0 = 98.1 \cdot 10^3$ N/m^2; T_g, gas temperature, °K; K_T, dimensionless correction coefficient; T, temperature, °C.

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MODELING OF IRREVERSIBLE PROCESSES BY ANALOGY METHODS OF STATISTICAL MECHANICS

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Analogies to statistical mechanics of a Gibbs ensemble are constructed on the basis of the reflection method for dissipative macroscopic processes.

The formalism of classical and irreversible thermodynamics can be used successfully to construct mathematical models of a broad class of different processes, particularly control processes [1]. This is primarily due to the existence of a profound analogy between the equations of a whole range of dynamic systems and thermodynamics. The analogy between nonequilibrium thermodynamics and analytical mechanics was analyzed in detail in [2, 3], where it was shown that thermodynamic relations can be represented in the form of Lagrange or Hamilton equations. The basis for this formalism is an artificial method of introducing the so-called "mirror reflected system" with negative dissipation and decreasing entropy [4].

The Lagrangian of the total system, including the original and reflected parts, can be represented in the form

$$\mathcal{L} = \tilde{K}(\dot{x}_i^*, \dot{x}_i) - \frac{1}{2}R(x_i^*, \dot{x}_i) + \frac{1}{2}R^*(\dot{x}_i^*, x_i) - \tilde{V}(x_i^*, x_i). \tag{1}$$

Here the functions $\tilde{K}$, $\tilde{V}$, R, R* are constructed on the basis of specific expressions for the kinetic and potential energies and the dissipation function.

By means of [1] one can introduce the generalized momenta

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \frac{\partial \tilde{K}}{\partial \dot{x}_i} - \frac{1}{2}\frac{\partial R}{\partial \dot{x}_i}; \quad p_i^* = \frac{\partial \mathcal{L}}{\partial \dot{x}_i^*} = \frac{\partial \tilde{K}}{\partial \dot{x}_i^*} + \frac{1}{2}\frac{\partial R^*}{\partial \dot{x}_i^*} \tag{2}$$

and the Hamiltonian

$$\mathcal{H} = p_i\dot{x}_i + p_i^*\dot{x}_i^* - \mathcal{L}. \tag{3}$$

Here and below doubly repeated subscripts imply summation. For an appropriate choice of the functions R and R*, $\mathcal{H}$ is an integral of motion.

For a system of material points, moving in a medium with a linear resistance law, the functions (1)-(3) are [4]

$$\mathcal{L} = a_{ik}\dot{x}_i^*x_k - \frac{1}{2}r_{ik}\dot{x}_i^*x_k + \frac{1}{2}r_{ik}\dot{x}_ix_k^* - b_{ik}x_i^*x_k, \tag{4}$$

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